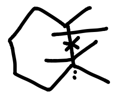
## I) Review

$B = \bigcup_{G \in P} G$ "base of SYZ" — $\mathbb{Z}$-polyhedra

$X \supset T^*B_0/\Lambda^*$
$\downarrow \qquad \downarrow$
$B \supset B_0 = B \setminus \Delta$

$\operatorname{codim} \Delta = 2$

$\Delta \subset |P^{[n-2]}|$



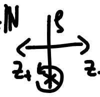
Aim: $X_s \subset \mathfrak{X}$, $\downarrow \quad \downarrow$, $s \in \operatorname{Spec} A$ $\quad A = \mathbb{C}[[t]]$ or $\mathbb{C}[[t]]/t^{k+1}$

$\Lambda \subset T_{B_0}$ $\mathbb{Z}$-vectors

Local models for $\mathfrak{X}$: $\dim G = n$ $\quad U_G = \operatorname{Spec} A[\Lambda_G]$

$\dim G = n-1$: $\quad U_g = \operatorname{Spec} A[\Lambda_g][z_+, z_-]/(z_+z_- - f_g t^c)$

$f_g \in A[\Lambda_g]$ $\quad t^c \in A$, $c \in \mathbb{N}$

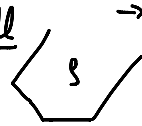


$R = \bigoplus A\theta_m$, $m$ asympt. monomial

$\mathring{\mathfrak{X}}' = \varinjlim_{g \subset G} (U_G \to U_g)$
$\downarrow$
$\operatorname{Spec} A$ $\quad$ need corrections to obtain fcts $\quad \Gamma(\mathring{\mathfrak{X}}, \mathcal{O}_{\mathring{\mathfrak{X}}})$

$\to$ wall structure

Wall



$\subset G$

$g \cap \operatorname{Int} G \neq \emptyset$ $\quad U_\mu = U_G$

$z^m \mapsto g_g^{\langle n_g, m\rangle} z^m$

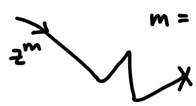
codim $g = 1$

$\Lambda_g^\perp = \mathbb{Z} \cdot n$

$\boxed{\text{Rem: } \mathfrak{X} \text{ affine} \Leftrightarrow B \text{ conical}}$

Canonical fcts on $\mathring{\mathfrak{X}}$

$\theta_m(p) = \sum_{\text{broken lines } \beta} a_\beta z^{m_\beta}$ + consistency $\rightsquigarrow$ well-defined



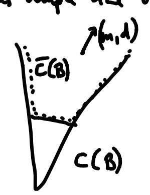
$m$ = asymptotic monomial
= asymptotic v.field on $B$

$\mathring{\mathfrak{X}}$ has ample line bdl $\mathring{\mathcal{L}}$

To get $\mathfrak{X} = \operatorname{Proj}(\dots)$:

$\operatorname{Tot}(\mathcal{L}^{-1})$

$S := \Gamma(\operatorname{Tot} \mathring{\mathcal{L}}^{-1}, \mathcal{O})$
$= \underbrace{\bigoplus_{d \geq 0} \bigoplus_{m \in B(\frac{1}{d}\mathbb{Z})} A \cdot \theta_{(dm, d)}}_{S_d}$

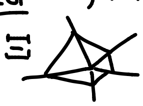


## II) Asymptotic monomials / LG-models

$S_0 = \bigoplus_{m \text{ asympt. monomial of } B} A \cdot \theta_m = R$

$\mathfrak{X} = \operatorname{Proj} S$
$\downarrow$ proper
$\mathcal{Y} = \operatorname{Spec} R$

LG: 1) Hori-Vafa asymptotically toric



toric $\quad \mathfrak{X} = \mathcal{Y} \quad \mathcal{Y} \xrightarrow{W} \mathbb{A}^1$

$W = \sum a_m z^m$

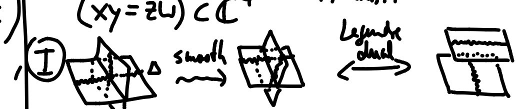
2) (Carl, Pumperla, S)

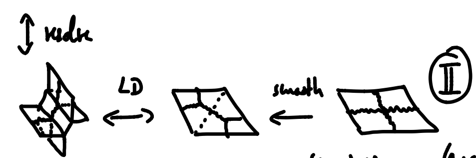
$\mathfrak{X} \xrightarrow{W} \mathcal{Y} = \mathbb{A}^1$

## III) Smoothy of conifolds

(w. Ruddat) (Malesi, C.-Bernard)

$(xy = zw) \subset \mathbb{C}^4$





$f_g \leftrightarrow$ Log structure
$f_g(s)$ $\quad s$ Parameter

I $\quad xy = (1+w)t^c \rightsquigarrow (1+sw)$
$\quad uv = (1+w)t^{c'} \quad (1+w)$

II $\quad uv = \underbrace{(1+x)(1+y)}_{\text{reducible}} \xrightarrow{s \neq 0}$ irreducible

$\exists\, \mathfrak{X}$ w. conifold points

II ✓ $\quad$ I ✓ (w. little work)

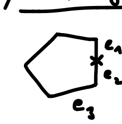
Claim $\quad \exists f_g(s)$

$H^1(B \setminus \{p_i\}, i_* \Lambda^*) \to H^2_{\{p_i\}}(B, i_* \Lambda^*)$ has good image

Friedman/Tian:

$\exists$ simult. smooth $\Leftrightarrow$ exc. curves of small resolution fulfil good relation

## IV) Variation of "kinks"



$N^{\# g} \ni (e_g) \leftrightarrow$ MPL fct

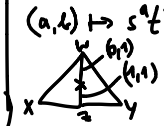
replace $\mathbb{N}$ by monoid $Q$

$\kappa_g \in Q$ monoid hom.

Example 1 $\quad \kappa_g : Q \to (A, \cdot)$

$Q = \mathbb{N}^2$, $A = \mathbb{C}[s, s^{-1}, t]$ $\quad xy = (sz + w)t$

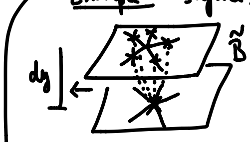
$(a, b) \mapsto s^a t^b$



$(1 + s\frac{z}{w})$

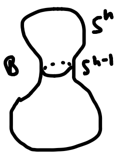
$(1 + s^{-1}\frac{w}{z})$

Example 2 surface sing.



$\tilde{X} \supseteq \mathbb{G}_m$
$\cup$
$(s=0) = X$

## Type II degeneration




$$\mathfrak{X} \longrightarrow \mathrm{Spec}(\mathbb{C}[s][[t]]) \longrightarrow \mathbb{A}^1_s$$

$$\mathfrak{X}_1 \cup_Y \mathfrak{X}_2 \qquad s=0$$

mirror

„stretching the neck"